WARM UP PROBLEM Find your copy of the Graphing Guidelines!

PRACTICE PROBLEMS

1. Sketch the curve $y = x - 2\sin x$ on $[-2\pi, 2\pi]$.

   (a) Find the domain. $\mathbb{R}$

   (b) Find the x and y-intercepts.

   when $x=0$, $y=0$.

   when $y=0$,... solve $2\sin x = x$? hard. let it go.

   Hey! once we are done we see there sure are other solutions. We'll find them in §4.8

   (c) Find the symmetries/ periodicity of the curve.

   x, $\sin x$ both odd.

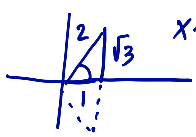
   So I expect the function to be odd.

   (d) Determine the asymptotes. none. $\lim_{x\to\infty} x-2\sin x = \infty$, $\lim_{x\to-\infty} x-\sin x = -\infty$.

   (e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 1 - 2\cos x = 0$

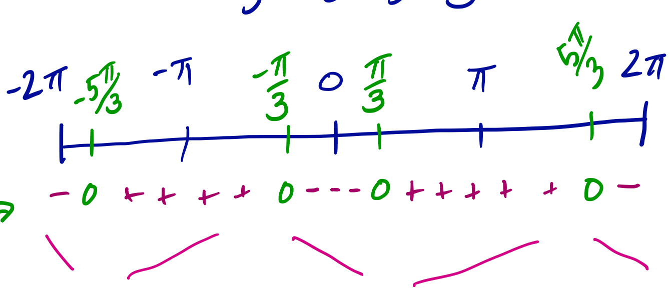
$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}, -\frac{\pi}{3}$

critical points in $[-2\pi, 2\pi]$ are:

$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$



Sign of y'

ANS:

y is increasing on $\left(-\frac{5\pi}{3}, -\frac{\pi}{3}\right) \cup \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

and decreasing on $\left(-2\pi, -\frac{5\pi}{3}\right) \cup \left(-\frac{\pi}{3}, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$.

local minimums at $x = -\frac{5\pi}{3}$, min value $-\frac{5\pi}{3} + \sqrt{3}$

at $x = \frac{\pi}{3}$, min value $\frac{\pi}{3} - \sqrt{3}$

at $x = 2\pi$, min value 2π

local maximums at $x = -2\pi$, max value -2π

at $x = -\frac{\pi}{3}$, max value $-\frac{\pi}{3} + \sqrt{3}$

at $x = \frac{5\pi}{3}$, max value $\frac{5\pi}{3} - \sqrt{3}$

(g) Find the intervals of concavity/inflection points.

$y' = 1 - 2\cos x$

So $y'' = 2\sin x$.

So $y'' = 0$ in $[-2\pi, 2\pi]$

when $x = -2\pi, -\pi, 0, \pi, 2\pi$

-2π		$-\pi$		0		π		2π	
0	+	0	−	0	+	0	−	0	← sign of y''

answer:

y is concave up on $(-2\pi, \pi) \cup (0, \pi)$ and

concave down on $(-\pi, 0) \cup (\pi, 2\pi)$.

inflection points:

x	$-\pi$	0	π
y	$-\pi$	0	π

(h) Sketch the curve.

points to plot:

$(-2\pi, -2\pi)$

$(-\frac{5\pi}{3}, \approx -3.5)$

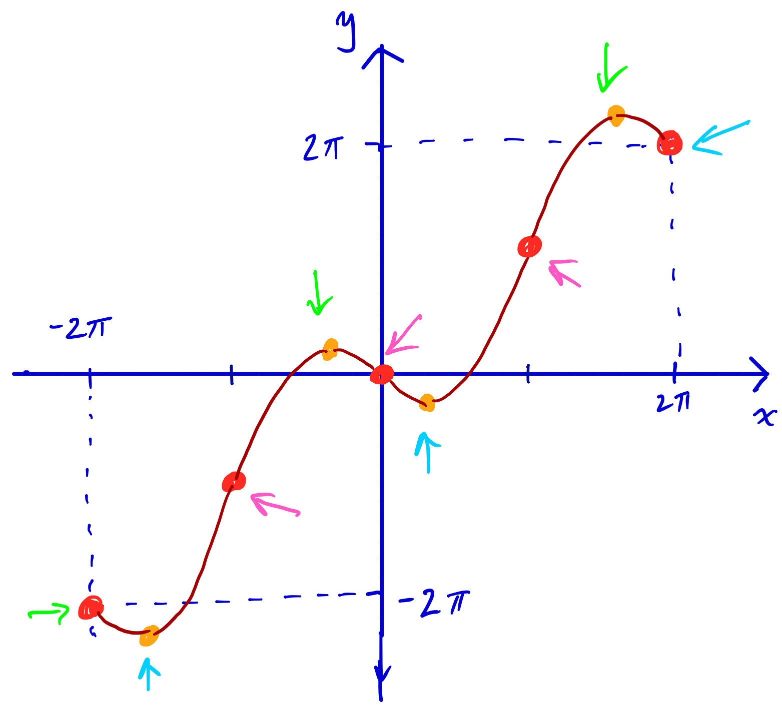
$(-\frac{\pi}{3}, \approx 0.69)$

$(0, 0)$

$(\frac{\pi}{3}, \approx -0.69)$

$(\frac{5\pi}{3}, \approx 3.5)$

$(2\pi, 2\pi)$



• local max pts

• local min pts

• inflection pts

2. Sketch the graph of $f(x) = \dfrac{3x^2}{x^2+4}$

(a) Find the domain. $\mathbb{R}$ (denominator never zero!)

(b) Find the x and y-intercepts.

$x=0, y=0.$

(c) Find the symmetries/ periodicity of the curve.

all terms are even. $f(x)$ is even.

(d) Determine the asymptotes.

$y=3$ since $\lim\limits_{x\to\pm\infty} f(x)=3.$ No vertical.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$f'(x) = \dfrac{24x}{(x^2+4)^2}$ * details on added page!

$x=0$ only critical pt.

denominator always positive.

$f'>0$ on $(0,\infty)$; $f'<0$ on $(-\infty,0)$

answer:

f is increasing on $(0,\infty)$ and decreasing on $(-\infty,0)$.

f has a local minimum at $x=0$ with minimum value $f(0)=0$.

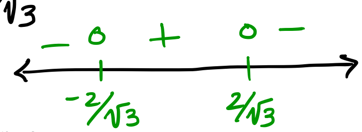
f has no local maximums.

(g) Find the intervals of concavity/inflection points.

$f''(x) = \dfrac{24(4-3x^2)}{(x^2+4)^3}$ * details on added page

$f''=0$ when $x=\pm 2/\sqrt{3}$

f'' never undefined.



answer

f is concave up on $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ and concave down on $(-\infty, -2/\sqrt{3}) \cup (2/\sqrt{3}, \infty)$.

Inflection points $\left(-2/\sqrt{3}, 3/4\right)$ and $\left(2/\sqrt{3}, 3/4\right)$

(h) Sketch the curve.

points to plot

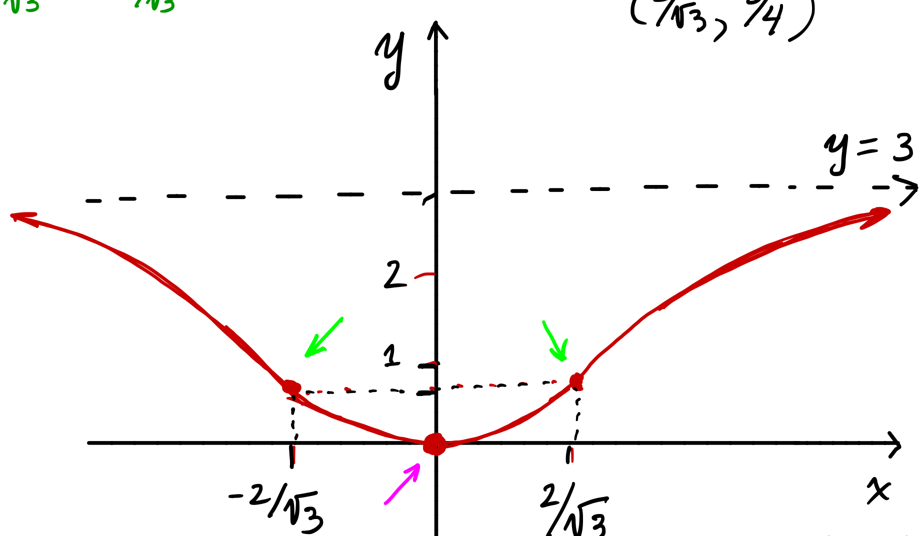
$(0,0)$

$\left(-\frac{2}{\sqrt{3}}, \frac{3}{4}\right)$

$\left(\frac{2}{\sqrt{3}}, \frac{3}{4}\right)$

• inflection points

• local min. pt.



* details

$$f(x) = \frac{3x^2}{x^2+4}$$

$$f'(x) = \frac{(x^2+4)(6x) - (3x^2)(2x)}{(x^2+4)^2} = \frac{6x^3 + 24x - 6x^3}{(x^2+4)^2}$$

$$= \frac{24x}{(x^2+4)^2}$$

#2 details

* details

$$f''(x) = \frac{(x^2+4)^2(24) - (24x)(2(x^2+4)(2x))}{(x^2+4)^4}$$

$$= \frac{24(x^2+4)[(x^2+4) - (x)(2)(2x)]}{(x^2+4)^4} = \frac{24[x^2+4-4x^2]}{(x^2+4)^3} = \frac{24(4-3x^2)}{(x^2+4)^3}$$

Inflection points

$$f(2/\sqrt{3}) = \frac{3 \cdot \frac{4}{3}}{\frac{4}{3}+4} \cdot \frac{3}{3} = \frac{12}{4+12} = \frac{12}{16} = \frac{3}{4}$$

3. Sketch the graph of $f(x) = x\sqrt{4-x^2}$

(a) Find the domain.

need $4-x^2 \geq 0$. So $-2 \leq x \leq 2$. ANS: $[-2,2]$

(b) Find the x and y-intercepts.

if $x=0$, $y=0$.

If $y=0$, $x = 0, +2, -2$.

(c) Find the symmetries/ periodicity of the curve.

even $\sqrt{4-x^2}$ multiplied by odd x gives odd. $f(x)$ is odd.

(d) Determine the asymptotes.

none

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$$f'(x) = \frac{2(2-x^2)}{\sqrt{4-x^2}}$$

$-2 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 2$

$- \; 0 \; + + + \; 0 \; -$ ← sign f'

$f'=0$ when $x = \pm\sqrt{2}$,

f'' undefined at $x = \pm 2$

answer: f increasing on $(-\sqrt{2}, \sqrt{2})$ and decreasing on $(-2,-\sqrt{2}) \cup (\sqrt{2},2)$.

f has local min at $x=-\sqrt{2}$, min value -2 and at $x=2$, min value 0.

f has local max at $x=\sqrt{2}$, max value 2 and at $x=-2$, max value 0.

(g) Find the intervals of concavity/inflection points.

$$f''(x) = \frac{2x(6-x^2)}{(4-x^2)^{3/2}}$$

$f''=0$ when $x=0, \cancel{\sqrt{6}}, -\cancel{\sqrt{6}}$ ← not in $[-2,2]$

f'' undefined at $x=-2,2$

$f''<0$ when $x<0$ and $f''>0$ when $x>0$.

answer: f is concave up on $(0,2)$ and concave down on $(-2,0)$.

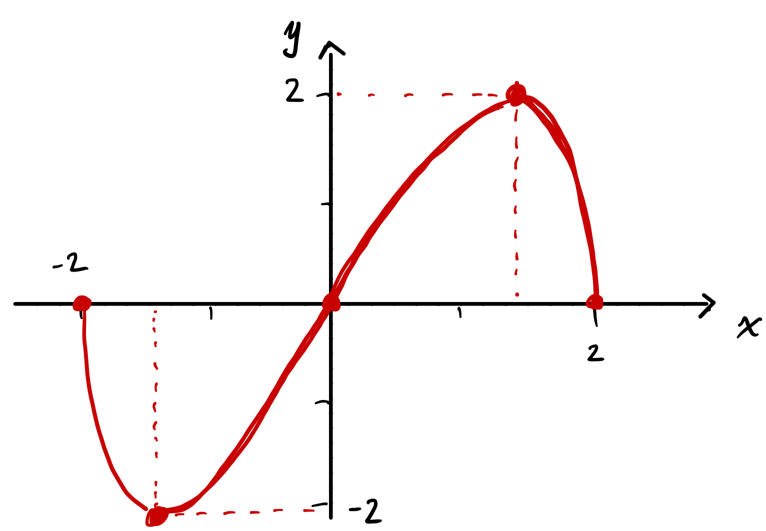
The point $(0,0)$ is an inflection point.

(h) Sketch the curve.

Points to plot

$(-2,0)$, $(0,0)$, $(2,0)$

$(-\sqrt{2},-2)$ $(\sqrt{2},2)$



details for example #3

$$f(x) = x(4-x^2)^{1/2}$$

$$f'(x) = 1\cdot(4-x^2)^{1/2} + x\cdot\frac{1}{\cancel{2}}(4-x^2)^{-1/2}\cdot(-\cancel{2}x)$$

$$= (4-x^2)^{1/2} - \frac{x^2}{(4-x^2)^{1/2}} = \frac{4-x^2-x^2}{(4-x^2)^{1/2}} = \frac{2(2-x^2)}{(4-x^2)^{1/2}}$$

get common denominator

$$f''(x) = \frac{(4-x^2)^{1/2}\cdot 2\cdot(-2x) - 2(2-x^2)\cdot\frac{1}{2}(4-x^2)^{-1/2}(-2x)}{(4-x^2)^{1}} = \frac{-4x\left[(4-x^2)^{1/2} - \frac{2-x^2}{2(4-x^2)^{1/2}}\right]}{4-x^2}\cdot\frac{2(4-x^2)^{1/2}}{2(4-x^2)^{1/2}}$$

$$= \frac{-4x\left[\overbrace{2(4-x^2) - (2-x^2)}^{8-2x^2-2+x^2=6-x^2}\right]}{2(4-x^2)^{3/2}} = \frac{-2x(6-x^2)}{(4-x^2)^{3/2}}$$

4. Sketch the curve $y = \dfrac{x}{\sqrt{9+x^2}}$

(a) Find the domain. $\mathbb{R}$

(b) Find the x and y-intercepts.

$(0,0)$

(c) Find the symmetries/ periodicity of the curve.

odd

(d) Determine the asymptotes. no vertical asymptotes.

$\lim_{x\to\infty} \dfrac{x}{\sqrt{9+x^2}} = 1$. So $y=1$ is a horizontal asym.

tricky! $\lim_{x\to-\infty} \dfrac{x}{\sqrt{9+x^2}} = -1$. So $y=-1$ is a horizontal asymp.

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$y' = 9(x^2+9)^{-3/2}$

So $y' > 0$ always.

answer: y is always increasing.
y has no local max's or mins.

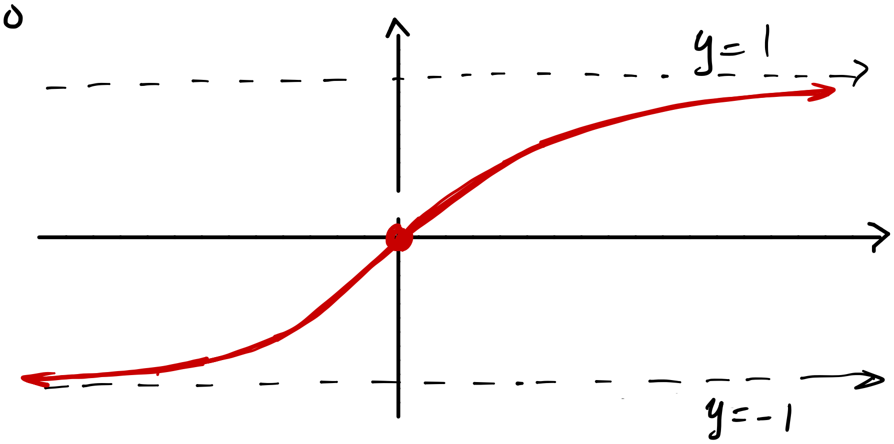
(g) Find the intervals of concavity/inflection points.

$y'' = \dfrac{-27x}{(x^2+9)^{5/2}}$

$y''=0$ when $x=0$.
$y''>0$ when $x<0$; $y''<0$ when $x>0$

answer y is concave up on $(-\infty,0)$ and concave down on $(0,\infty)$.
The point $(0,0)$ is an inflection point.

(h) Sketch the curve.



5. Sketch the curve $y = \dfrac{x^3+4}{x^2}$

(a) Find the domain. $(-\infty, 0) \cup (0, \infty)$

(b) Find the x and y-intercepts.

no y-intercept

Set $y=0$. Then $x = \sqrt[3]{-4} \approx -1.587$

(c) Find the symmetries/ periodicity of the curve. none

the x^3+4 destroys all hope

(d) Determine the asymptotes. (Try to find the slant asymptote. That is, what *line* does this function approach as $x \to \pm\infty$?)

$x=0$ vertical asymptote.

$\lim_{x\to\infty} \frac{x^3+4}{x^2} = \lim_{x\to\infty} x + \frac{4}{x^2}$ which should get closer and closer to $y=x$.

Slant asymptote: $y=x$

(e,f) Determine where the function is increasing/ decreasing and find the local maximum/ minimum values

$$+ \quad 0 \quad - \quad 2 \quad +$$

$y' = 1 - \frac{8}{x^3}$

$y' = 0$ when $x=2$

y' undefined when $x=0$

ans: y is increasing on $(-\infty,0)\cup(2,\infty)$ and decreasing on $(0,2)$

y has a local min at $x=2$ with min value 3

(g) Find the intervals of concavity/inflection points.

$y'' = 24x^{-4} = \frac{24}{x^4}$, which is positive where it is defined.

Ans: y is concave up on $(-\infty,0)\cup(0,\infty)$ with no inflection points.

(h) Sketch the curve.